http://dx.doi.org/10.15576/GLL/2014.2.75

GLL

Geomatics, Landmanagement and Landscape No. 2 • 2014, 75–85

THE IMPORTANCE OF THE OBJECTIVE FUNCTIONS AND FLEXIBILITY ON CALIBRATION OF PARAMETERS OF CLARK INSTANTANEOUS UNIT HYDROGRAPH

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Summary

The paper compares the results of automatic calibration of Clark's synthetic unit hydrographs. Optimal values of model parameters were determined by the objective functions: percentage error in peak flow (PEPF), percentage error in volume (PEV), peak-weighted root mean square error (PWRMSE), sum of absolute residuals (SAR) and sum of squared residuals (SSR). The last part of the analysis assesses the flexibility of studied model. The research was performed in the upland watershed of Grabinka – left tributary of the Wisłoka river located in Southern Poland. The analysis reveals that the smallest differences between the maximum flow in the observed and calculated flood culmination were obtained when applying PWRMSE function. This paper also indicates, that Clark's model was efficient for describing the analyzed floods.

Keywords

flood wave • calibration • elasticity • modeling • rainfall

1. Introduction

The parameters of a hydrological model are estimated in the calibration process, which involves minimization of variation between the observed and calculated values [Henrichs et al. 2008]. The previous "manual" calibration technique, which involved adjusting model parameters and verifying the simulation results, has been replaced with algorithms enabling automation of the process, e.g. Monte Carlo method [Bahremand and De Smedt 2008, Di Pierro 2005, Papadopoulos and Yeung 2001]. The model is considered to be effective, if the calibration process provides a set of parameters that will eventually allow obtaining the result of simulation that is similar to the observation. Selection of automatic calibration algorithm depends on the criterion adopted for qualitative assessment of the model. The first stage of the process is the analysis of model errors. In practice, the applied methods involve comparing the observed and calculated values. The resulting measurement error of the calibration process is necessary to evaluate the model – it reflects the quality of model results' adjustment to the observation. One can distinguish two basic measures: mean square error (MSE) and sum of squared residuals (SSR) [Kamali 2009]. Another common measure used in hydrology is the Nash-Sutcliffe efficiency coefficient, a normalized MSE, which is defined as [Nash and Sutcliffe 1970]:

$$E = 1 - \frac{MSE}{s_o^2}, \tag{1}$$

where:

s_o – standard deviation of observed discharges.

The next stage of the calibration process is a sensitivity analysis. The sensitivity analysis allows examining the interactions between model parameters and obtained simulation results. This analysis enables, among others, to determine the contribution of individual parameters or combinations thereof in the final outcome of the simulation. Automatic calibration process is frequently supplemented by a selection of algorithm to optimize the parameters. This is a very important phase of calculations as, if properly completed, it allows obtaining optimal parameter values and thus achieving the best adjustment of the model to the actual course of the analyzed phenomenon.

Unfortunately, the need to conduct a full process of model calibration in practice is often forgotten. In effect, the results obtained from the calculations do not always properly reflect the reality. Often the calibration process is based primarily on the global assessment of quality of model results' adjustment to the observations using a single measure. In practice, algorithms for the calculation of different objective functions and searching for their minimum at which the most optimal set of parameters is obtained are well recognized. On the other hand, there is not enough research that compares different measures of model quality and evaluates their usefulness in the calibration process.

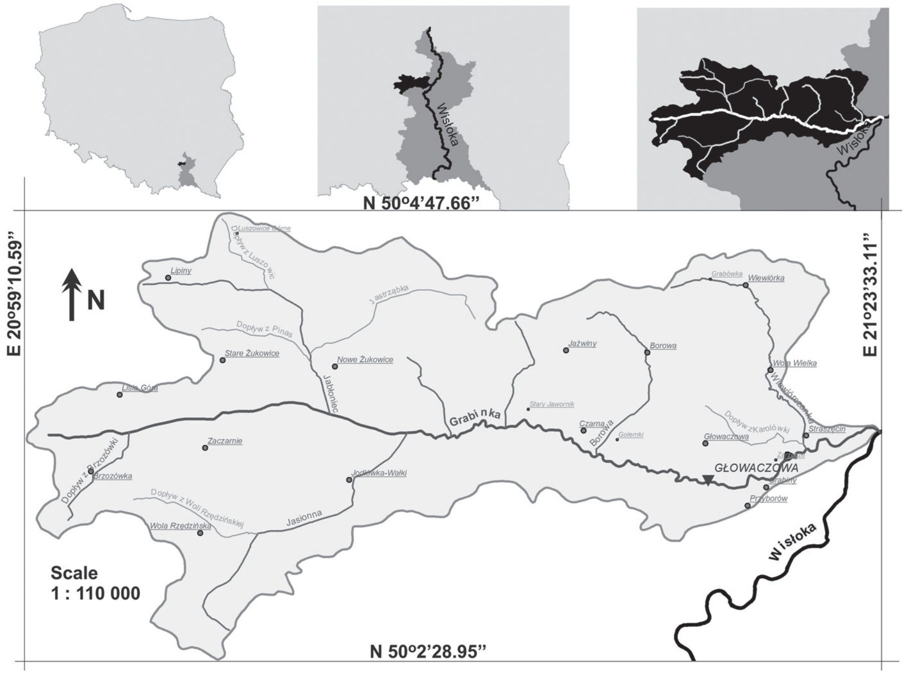
Sensitivity analysis results can be used to decide on which parameters the model calibration efforts should focus, or even as an analysis tool to test if the model behaves according to underlying assumptions [Castings et al. 2009]. Ultimately, sensitivity methods should serve as diagnostic tools that help to improve mathematical models and potentially help us to identify gaps in our knowledge that are most severe and affect prediction uncertainty the most. The characteristics of different methods for model sensitivity analysis (for example Sobol analysis, Kullback-Leibler entropy, Morris method or regression analysis) are describes by Pappenberger et al. [2008]. Model sensitivity analysis allows assessing its uncertainty, which is the result of data input errors as well as improper model structure and errors in determining its parameters [Sikorska et al. 2012].

One of the methods of model sensitivity analysis is the determination of its flexibility. Even though this method allows assessing the impact of model input parameters on the results of calculations, it has not been widely adopted. It has been described only in the paper [Maidmend and Hoogerwerf 2002], where it has been used in the analysis of an NRCS-UH (National Resources Conservation Service – Unit Hydrograph) model.

The aim of this paper was to evaluate the impact of the calibration process, using a variety of objective functions, on the accuracy of the phenomenon's description by the model. The Clark Instantaneous Unit Hydrograph models were used in the analysis of discharge floods in the upland watershed. Choosing the objective function significantly affects the efficiency of the applied model. The study used a novel approach to calibrate hydrological models, taking into account also a sensitivity analysis using the coefficient of elasticity in addition to the minimization of the objective function itself.

2. The study area

The analyses were performed in the watershed of Grabinka – the left secondary tributary of the Wisłoka river (Figure 1) located in the southern part of Poland (Podkarpackie voivodeship). The area of the watershed is 218.68 km^2, the average watershed slope equals 5.46‰, and the length of the main watercourse is 32.82 km. In the analyzed watershed quaternary deposits lay on the Miocene clays: sands with boulders, boulder clays and fluvial sands (Podział hydrograficzny Polski 1983). The average annual rainfall in the watershed is approximately 650 – 700 mm [Lorenc 2005]. The watershed is dominated by permeable soils and the land cover is mostly represented by agricultural land and forests.



Source: author's study

Fig. 1. Watershed of Grabinka

3. Material and methods

The analysis was based on the highest daily precipitation recorded at the Tarnów precipitation station and the corresponding flood hydrographs observed in the cross-section of Głowaczów water gauge, which closes the Grabinka watershed. The analysis was based on the episodes of 1980, 1981, 2004 and 2006. The selection of data for the analyses was caused by their availability. These data originated from the archives of Institute of Meteorology and Water Management, National Research Institute.

Due to the availability of point-only precipitation data, before the analysis has started, it was transformed into precipitation distributed on the watershed area. Transformation was based on the precipitation reduction curves as functions of duration and watershed area, presented by Ponce [1989]. The division of total hydrograph into direct and basic discharge was based on a recession curve. Effective precipitation, which describes direct discharge, was determined by SCS (Solid Conservation Service) method. In the presented study the value of CN (Curve Number) parameter was determined by optimization using the observed rainfall-discharge phenomenon. The Clark's model based on synthetic unit hydrographs was subjected to the calibration process. Clark developed a method for generating a watershed's unit hydrograph that is based on the relationship between time of concentration and the watershed area, and uses a theory of a single linear reservoir to transform effective precipitation into discharge. Treating the watershed as a linear reservoir allows for including the phenomenon of retention in the watershed. Clark's method is an attempt to link geo-morphological characteristics of the watershed with its reaction to precipitation [Cleveland et al. 2008]. In Clark's method the watershed retention described by a single linear reservoir is expressed by the equation:

$$S = R \cdot O \tag{2}$$

where:

S – the volume of retention in the watershed,
R – watershed retention coefficient,
O – volume of discharge from the watershed.

In the first stage of the calculations, model parameters were automatically calibrated based on the following measures (objective functions): percentage error in peak flow (PEPF), percentage error in volume (PEV), peak-weighted root mean square error (PWRMSE), sum of absolute residuals (SAR) and sum of squared residuals (SSR). All characteristics are specified with the equations [Cunderlik and Simonovic 2004]:

$$\mathrm{PEPF} = 100\left[\frac{Q_O - Q_S}{Q_O}\right] \tag{3}$$

$$\mathrm{PEV} = 100\left[\frac{V_O - V_S}{V_O}\right] \tag{4}$$

$$\mathrm{PWRMSE} = \sqrt{\frac{\sum_{t=1}^{N} (Q_O(t) - (Q_S(t))^2 \cdot \frac{Q_O(t) + Q_{\mathrm{ave}}}{2Q_{\mathrm{ave}}}}{N}}, \; Q_{\mathrm{ave}} = \frac{1}{N}\sum_{t=1}^{N} Q_O(t) \tag{5}$$

$$\mathrm{SAR} = \sum_{t=1}^{N} \left| Q_O(t) - Q_S(t) \right| \tag{6}$$

$$\mathrm{SSR} = \sum_{t=1}^{N} [Q_O(t) - Q_S(t)]^2 \tag{7}$$

where:

$Q_O(t)$ and $Q_s(t)$ – observed and simulated flow in time t,
Q_{ave} – average observed flow,
V_O and V_s – volumes of observed and simulated wave.

Automatic calibration of model parameters was based on iterative selection of the parameters until the minimum of the objective function. In order to minimize the objective function (3) to (7) a uniform gradient method was applied. This involves estimating the value of one parameter while maintaining the remaining stable.

The final evaluation of the calibration process was based on Nash-Sutcliffe efficiency coefficient E [1970], commonly used in hydrology:

$$\mathrm{E} = \left[1 - \frac{\sum_{i-1}^{i=N} (Q_O - Q_S)^2}{\sum_{i-1}^{i=N} (Q_O - \overline{Q_O})^2} \right] \tag{8}$$

where:

N – the number of hydrograph ordinates,
i – the index changing from 1 to N,
Q_O – the ith ordinate of the observed hydrograph,
Q_s – the ith ordinate of the simulated hydrograph,
$\overline{Q_O}$ – the average of the observed hydrograph ordinates.

After the model calibration process, its sensitivity to changing parameters was analyzed. Sensitivity of a model was characterized by its flexibility, which is a measure of impact of one parameter on another. It is a non-unitary parameter, which is calculated as the ratio of the percentage change in the output characteristics to the percentage change in input parameter [Maidment and Hoogerwerf 2002]. If values of this parameter are higher or equal to 1, then the parameter is “flexible”; in other words, the dependent variable is very sensitive to the size of independent variable. Otherwise, when flexibility is lower than 1, the parameter is “inflexible” and the dependent variable is not sensitive to the change of independent variable. The aim of the sensitivity analysis was to determine the effect of the time of concentration and retention coefficient in Clark’s model on the variability of culmination flow in a simulated hydrograph. The analysis consisted on setting different values of parameters and calculating Q_{max} flow. Based on such analyses it was possible to determine the flexibility of a given parameter.

4. Results and discussion

4.1. Calibration of parameters

Calculations have shown that by using the objective function described by formula (3) one can obtain complete consistency between culminations of calculated and observed waves (Table 1). This is a typical feature of this statistic, where the only criterion for searching the model parameter values is to minimize the difference between these culminations [Cunderlik and Simonovic 2004]. A similar principle shall apply in the case of the other measure – PEV, except that in this case, the model tends to minimize differences in the volume between the compared waves. Unfortunately, adopting these criteria has negative impact on the shape of the calculated wave.

Table 1. Values of objective functions for the waves described by Clark’s model

Wave	PEPF [%]	PEV [%]	PWRMSE [%]	SAR [$m^3 \cdot s^{-1}$]	SSR [–]
1980	0.0 (0.0)*	0.0 (14.7)	8.6 (–13.5)	61.2 (–30.9)	819.7 (–34.4)
1981	0.0 (0.0)	0.0 (–45.1)	4.7 (–13.0)	42.0 (–86.8)	423.9 (–49.8)
2004	0.0 (0.0)	0.0 (–7.1)	1.0 (0.8)	28 (–3.4)	29.7 (–1.1)
2006	0.0 (0.0)	0.0 (–68.5)	5.3 (–23.4)	76.7 (–37.7)	562.5 (–35.2)

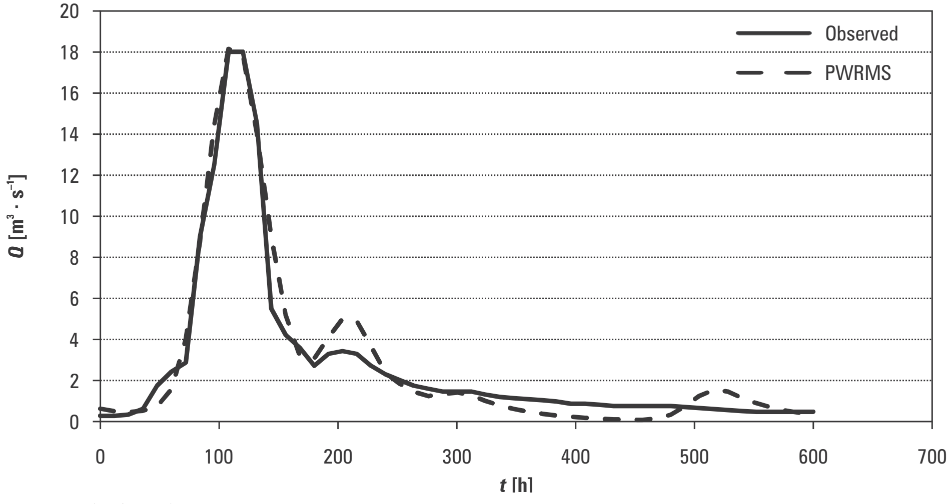
* In brackets are percentage difference between peak flows in observed and simulated wave

Source: author’s study

The best calibration results were obtained using PWRMSE (omitting the PEPF function). Figure 2 compares the observed waves of 2004 and waves obtained from Clark’s model using automatic calibration with PWRMSE. The smallest difference between calculated and observed Q_{max} was obtained for the episode of 2004 and the worst results (the highest values of objective functions) – for the wave of 2006. The worst results of the simulations were obtained when the model parameters were optimized using SAR; the difference between Q_{max} calculated and observed reached almost – 87%.

Similarly to PEPF, null values were obtained also in the second criterion (PEV), but the shapes of simulated hydrographs considerably differed from the observed hydrographs. Moreover, apparent disparity between Q_{max} in culminations between observed and calculated waves were observed from about –7% to over –68%. For Clark's model lower values of objective functions were observed for the waves of 1981 and 2004. In most cases the value of Q_{max} was underestimated. The most common cause for generating the largest errors in calibration was inconsistency between the flow of culmination and different course of the ascending part of calculated and simulated wave (more rapid increase of calculated wave).

Flow residues defined as the differences between the calculated and observed instantaneous flow, where the objective function is described by the equations from (3) to (7). The course of residues is similar for either of the objective functions. Times to culminations of calculated waves in all analyzed cases coincide with the time to culmination of the observed wave. Positive values indicate that for calculated waves flows were higher than for the observed waves. This is due to the adopted model of effective precipitation, which determines the center of gravity when calculating the concentration and lag time [Sikorska and Banasik 2008]. Minor errors are generated in the zone of flow recession curve. Undoubtedly, the initial values of variables play a significant role in the correctness of calibration procedure and thus affect the size of errors generated by the model [Velez and Frances 2005].



Source: author's study

Fig. 2. Comparison of the observed wave of 2004 with the wave calculated from Clark's model using calibration with PWRMSE

4.2. Quality of models

The next stage of the calibration of model parameters is the general assessment of the quality, which was performed using the coefficient of efficiency E. The results are presented in Table 2.

Table. 2. Values of the coefficient of efficiency E [%] for each objective function and analyzed

Wave	Clark IUH				
	PEPF	PEV	PWRMSE	SAR	SSR
1980	24.9	39.7	39.6	49.4	49.4
1981	37.2	54.8	45.2	35.3	52.5
2004	82.3	89.1	95.9	95.8	96.6
2006	0.8	–13.7	80.2	83.8	82.8

Source: author's study

The lowest efficiency of model was obtained when parameters were optimized using PEV and PEPF functions. Generally, when comparing both objective functions, slightly better results were obtained by optimizing the model parameters using PEPF. Approximated values of the coefficient of efficiency E were obtained when optimization of model parameters was performed using the other objective functions. This follows from the fact that the error values are calculated for the entire course of the hydrograph and not for its selected parts. Generally, high values of coefficient E in optimizing the parameters with PWRMSE, SAR and SSR were obtained for two waves: of 2004 and 2006. Similarly, high efficiency of Clark's model was showed by Straub et al. [2000] when analyzing IUH (Instantaneous Unit Hydrograph) in small agricultural watersheds in Illinois, U.S. Verification of this model revealed that in 21 of 29 analyzed watersheds the flow error did not exceed 25%. Values of coefficient of efficiency E in the vast majority exceeded 90%.

4.3. Flexibility analysis

The final element of the model parameter calibration was the flexibility analysis.

Clark's model is inflexible to the change of the following parameters: T_c – time of concentration and R – retention factor. The calculated coefficient of flexibility for T_c was – 0,26 and for R – 0.74. As a result of increasing time of concentration the size of flows is gradually reduced and the hydrograph is postponed (Figure 3a). The increase in concentration time extends the reaction of watershed to precipitation, which delays discharge and slightly reduces flows. Increasing the retention factor R also contributes to reducing the size of flows, but without changing the shape of the hydrograph. The lower the value of this parameter, the greater the amount of precipitation is collected in the watershed area and thus the discharge increases (Figure 3b). The analysis revealed that Clark's model is significantly more susceptible to the change of the R parameter, than to the change of concentration time. These results coincide with those observed by Ahmad et al. [2009]. According to these authors the phenomenon of discharge wave diffusion in Clark's model dominates over the effect of discharge delay.

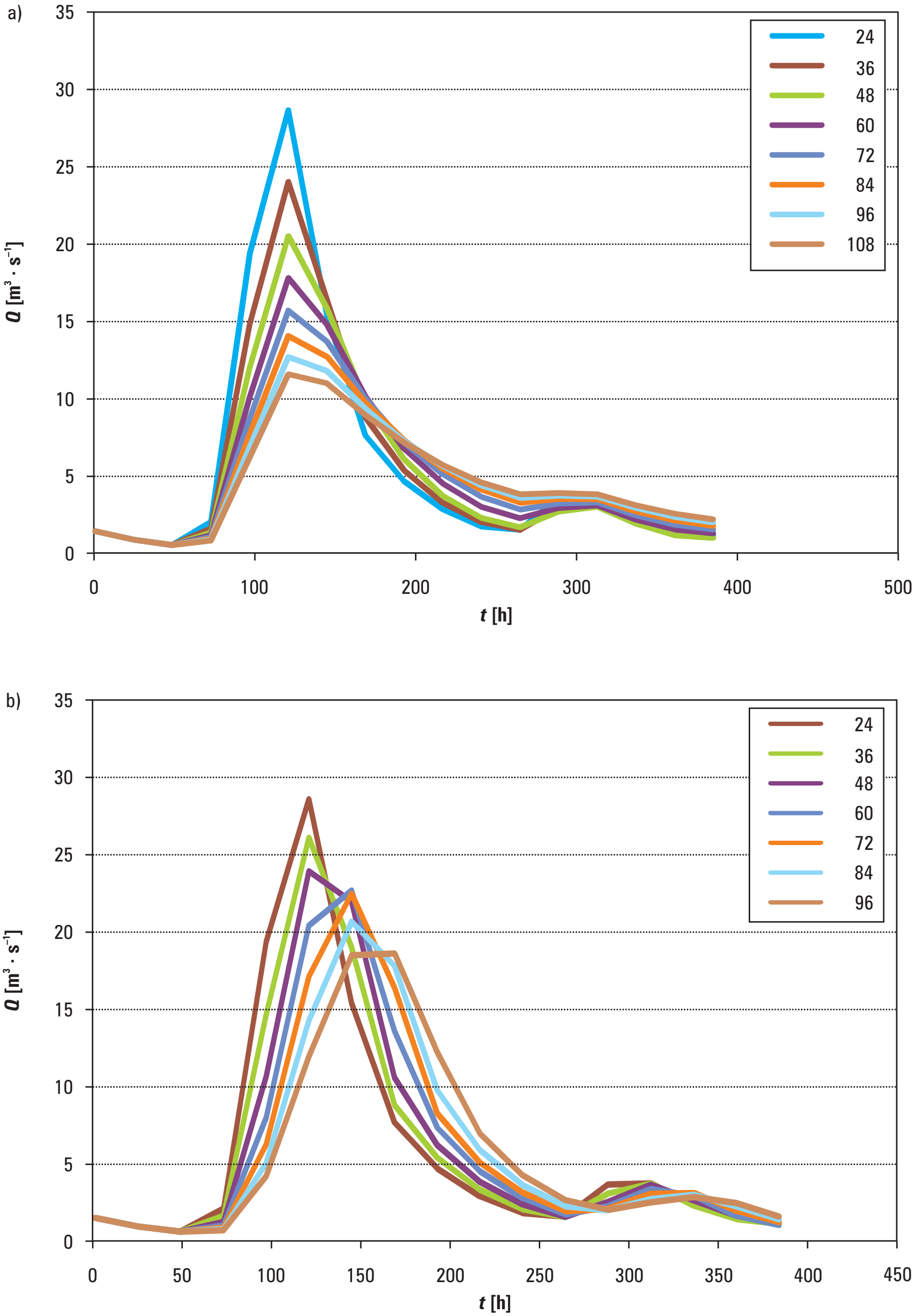


Fig. 3. Effect of the change of time of concentration T_c (a) and retention factor R (b), on the shape of discharge hydrograph in Clark's model

5. Conclusions

The following conclusions were reached as the result of conducted analyses:

1. The smallest differences between the maximum flow in the culmination of observed and calculated waves were obtained using PWRMSE. This follows from the fact that in the optimization procedure higher weights were assigned to the errors of flows located closer to the culmination flows. This measure should be used for the calibration of model parameters.
2. The weakest calibration effects were obtained using PEPF and PEV measures. This is caused by the fact that the sole criterion for searching of model parameter values is to minimize the difference between culminations and volumes of calculated and observed wave.
3. Considering the coefficient of efficiency *E*, Clark's model was efficient in the description of the analyzed floods This is due to the fact that transformation of effective precipitation into discharge in Clark's model is carried out regarding the retention process in the watershed and translation in the watercourse bed. This indicates that Clark's model may be applied in precipitation flood simulations in upland watersheds.
4. Clark's model is inflexible to the change of both its parameters – the time of concentration and the retention coefficient.
5. Sensitivity analysis is a novel approach to the analysis of uncertainty related to hydrological models. Its inclusion in the calculation will reduce the error generated by the model with the correct estimation of the key parameters which will influence the accuracy of the description of reality by model.

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