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THE PROPOSITION OF A NEW DAMPING FUNCTION FOR OUTLIERS IN THE ADJUSTMENT PROCESS

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Summary

This study proposes of new damping function in the robust estimation process. The formulae for a behaviour of the damping function were provided and a diagram of adjustment process was presented. The numerical examples are given in order to assess the efficiency of the proposed computational algorithm in accomplishing a typical geodetic task with outliers or gross errors.

Keywords

outliers • robust estimation • dumping function • conic curve

1. Introduction

The adjustment of geodetic surveys is usually made by the standard least-square adjustment method (LSQ). It belongs to the wide range of the so-called M-estimation methods [Wiśniewski 2005]. LSQ is called a neutral method of surveying adjustment, in which random errors (including gross errors) are dispersed in a surveying system on condition that the residual sum of squares is minimal. Thanks to it the results without a gross error can be contaminated and that will have negative influence on determined parameters (e.g. coordinates). Among M-estimation methods a group of robust estimations can be singled out [Kadaj 1980, 1988, Kamiński and Wiśniewski 1992, Wiśniewski 2008, Xu 2005], aimed at detection and elimination of outliers.

One of the ways of carrying out a robust estimation consists in modification of weight function as a goal function in the LSQ method, that is in introduction of the so-called damping function in the adjustment process [Wiśniewski 2005]. Among the most popular damping functions are that of Hampel's [1971], Huber's [1964], Danish [Prószyński and Kwaśniak 2002], and the functions proposed by the author of this paper: the QDF [Gargula 2007] and EDF methods [Gargula and Krupiński 2007, Gargula 2010]. Two examples of these functions are shown in Figure 1.

Modification of the LSQ method consists in multiplying the initial weights p_i, or diagonal elements of weight matrix P, by adequate damping indices ($\bar{v}_i$):

$$\boldsymbol{P} = \mathrm{diag}\{p_i;\quad i = 1,\ 2,\ \ldots,\ n\} \tag{1}$$

$$\hat{p}_i = p_i \cdot f(\bar{v}_i) \tag{2}$$

where:

n – number of observations,

$\hat{p}_i$ – modified weights.

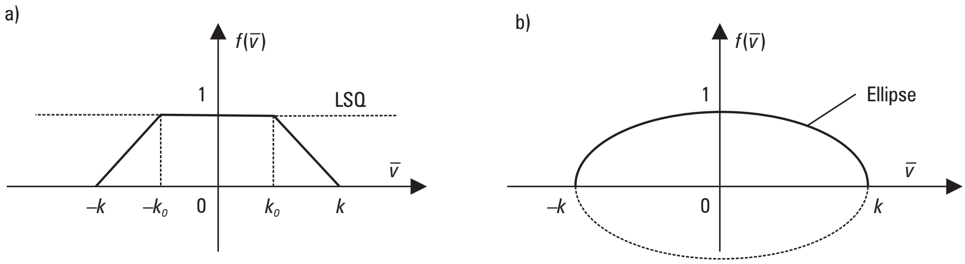

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Fig. 1. Examples of damping functions: the Hampel's function [1971], the 'elliptic' function (EDF) [Gargula and Krupiński 2007] (LSQ – least-square adjustment method, $\bar{v}$ – standardized residual, $f(\bar{v})$ – damping function, k, k_o – control parameters of damping functions)

The damping indices $f(\bar{v})$ are calculated for the determined standardized residual values $\bar{v}_i$. Standardisation involve the initial residual v_i, that is the residual obtained by the LSQ method (first step of robust estimation):

$$\bar{v}_i = \frac{v_i}{\sqrt{Q_{ii}}} \tag{3}$$

The indices Q_{ii} are diagonal elements of variance-covariance matrix of residual vector V:

$$\mathbf{Q_v} = \mathbf{P}^{-1} - \mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{PA})^{-1}\mathbf{A}^{\mathrm{T}} \tag{4}$$

where: A – matrix of indices of residual equation system

After calculating new weights (2) another stage (iteration) of robust estimation follows, that is residual estimation by the LSQ method with using modified weight matrix.

The proposition of a new damping function will consist in introducing some changes in the behaviour of the EDF function (Figure 1b) with the aim of 'smoothing' the passage from the range of damping the observations <–k, k> to the range of elimination (rejecting) of observations $|\bar{v}_i| > k$.

2. The proposed damping function

The general equation of conic curve (ellipse) of semi-axis length a, b can be written in the coordinate system as presented in Figure 1b [Bronsztejn and Siemiendiajew 1990]:

$$\frac{\overline{v}^2}{a^2} + \frac{f(\overline{v})^2}{b^2} = 1 \tag{5}$$

On the assumption that $a = k$ and $b = 1$, the equation of damping function EDF (Figure 1b) can be written as follows [Gargula and Krupiński 2007]:

$$f = \begin{cases} 1 & for \quad \overline{v}_i = 0 \\ f(\overline{v}_i) = \sqrt{1 - \frac{\overline{v}_i^2}{k^2}} & for \quad \overline{v}_i \in \langle -k; k \rangle \\ 0 & for \quad |\overline{v}_i| > k \end{cases} \tag{6}$$

The damping function proposed in this study is a spline, consisting of a fragment of conic curve and two tangents at points k_o and $-k_o$. In Figure 2 two examples of functions f_1 and f_2 are shown, differing by tangents points ($k_{o(1)}$, $k_{o(2)}$) of an ellipse and a line segment. For the purposes of this study the working notation: ELDF (elliptical and linear damping function) has been suggested. The efficiency of appropriate dampening influence on the outliers will depend on the behaviour of the proposed function.

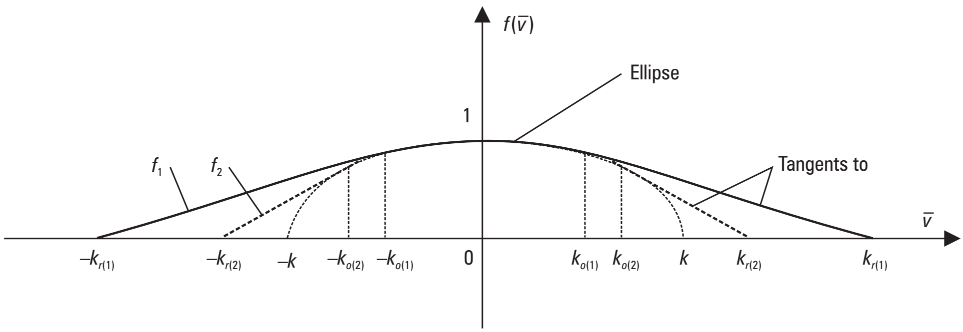

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Fig. 2. The proposed damping functions (k, k_o, k_{or} – control parameters of damping function)

The initial equation of a tangent line to an ellipse at a point k_o can be expressed as below [Bronsztejn and Siemiendiajew 1990]:

$$f(\overline{v}) - f(\overline{v} = k_o) = f'(\overline{v} = k_o) \cdot (\overline{v} - k_o) \tag{7}$$

$$f(\overline{v} = k_o) = \sqrt{1 - \frac{k_o^2}{k^2}} \tag{8}$$

$$f'(\overline{v} = k_o) = -\frac{k_o}{k^2\sqrt{1-\frac{k_0^2}{k^2}}} \tag{9}$$

where:

$f(\overline{v} = k_o)$ – value of EDF function (6) at a point k_o;
$f'(\overline{v} = k_o)$ – a derivative of this function for $\overline{v} = k_o$.

Therefore the ultimate form of the tangent equation will be as follows:

$$f(\overline{v}) = -\frac{k_o}{k^2\sqrt{1-\frac{k_0^2}{k^2}}} \cdot \overline{v} + \frac{k_0^2}{k^2\sqrt{1-\frac{k_0^2}{k^2}}} + \sqrt{1-\frac{k_0^2}{k^2}} \tag{10}$$

In order to determine the intersection point k_r of a tangent to horizontal axis (Figure 2), that is the point of rejection of outliers from the adjustment process, the equation (10) would be transformed for a value of a function $f(\overline{v} = k_r) = 0$. The result would be:

$$k_r = \frac{k^2}{k_o} \tag{11}$$

Analogically to (10), one gets an equation of a tangent at a point $\overline{v} = -k_o$:

$$f(\overline{v}) = \frac{k_o}{k^2\sqrt{1-\frac{k_0^2}{k^2}}} \cdot \overline{v} + \frac{k_0^2}{k^2\sqrt{1-\frac{k_0^2}{k^2}}} + \sqrt{1-\frac{k_0^2}{k^2}} \tag{12}$$

At the end of this derivation a full notation of the proposed damping function (ELDF) can be compiled as follows:

$$f = \begin{cases} 1 & for \quad \overline{v}_i = 0 \\ f(\overline{v}_i) = \sqrt{1-\frac{\overline{v}_i^2}{k^2}} & for \quad \overline{v}_i \in \langle -k_o ; k_o \rangle \\ f(\overline{v}_i) = -\left| \frac{k_o}{k^2\sqrt{1-\frac{k_0^2}{k^2}}} \cdot \overline{v}_i \right| + \frac{k_0^2}{k^2\sqrt{1-\frac{k_0^2}{k^2}}} + \sqrt{1-\frac{k_0^2}{k^2}} & for \quad |\overline{v}_i| \in (k_o ; k_r \rangle \\ 0 & for \quad |\overline{v}_i| > k_r \end{cases} \tag{13}$$

From the damping function equation (13) it directly results the following notation of weight function:

$$p(\overline{v}_i)=f\cdot p_i=\begin{cases} p_i & for\ \overline{v}_i=0 \\ \sqrt{1-\frac{\overline{v}_i^2}{k^2}}\cdot p_i & for\ \overline{v}_i\in\langle -k_o;k_o\rangle \\ \left(-\left|\frac{k_o}{k^2\sqrt{1-\frac{k_0^2}{k^2}}}\cdot\overline{v}_i\right|+\frac{k_0^2}{k^2\sqrt{1-\frac{k_0^2}{k^2}}}+\sqrt{1-\frac{k_0^2}{k^2}}\right)\cdot p_i & for\ |\overline{v}_i|\in(k_o;k_r\rangle \\ 0 & for\ |\overline{v}_i|>k_r \end{cases} \tag{14}$$

Later in this study the derived equations (13), (14) will be tested on practical examples.

3. Numerical examples – calculation plan

Calculation examples would refer to two forms of the proposed damping functions (Figure 2):

- the function f_1 for $k_{0(1)} = 0.5k$ (the method would be described as ELDF1),
- the function f_2 for $k_{0(2)} = 0.7k$ (ELDF2).

The equation of the proposed damping function, expressed in a general form (10) for a definite value of the parameter k_0 takes the following form:

$$f_1(\overline{v}) = -\frac{0.5}{\sqrt{0.75}\cdot k}\cdot\overline{v}+\frac{1}{\sqrt{0.75}} \tag{15}$$

$$f_1(\overline{v}) = -\frac{0.5}{\sqrt{0.75}\cdot k}\cdot\overline{v}+\frac{1}{\sqrt{0.75}} \tag{16}$$

The value of the boundary parameter k_r, denoting rejection of observations (see Figure 2), would be adequately:

$$k_{r(1)} = 2k \tag{17}$$

$$k_{r(1)} = \frac{10}{7}k \tag{18}$$

To check the suggested damping functions the example of adjustment of simple levelling net (junction point in levelling net) was used, as presented in Figure 3. Initial data are included in Table 1.

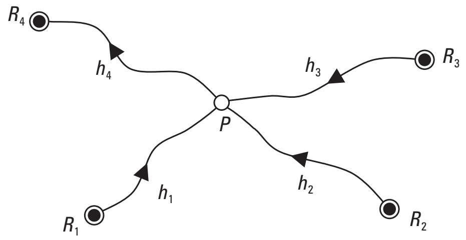

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Fig. 3. Test levelling net (R_i – benchmarks, P – calculated point, h_i – measured height differences)

Table 1. Initial data for numerical example

Nr (i)	Benchmark height R_i [m]	Height differences h_i [m]	Mean calculation errors m_i [m]	Approximate height of point P ($H'_i = R_i + h_i$) [m]
1	214.000	0.991	0.004	214.991
2	216.000	−1.002	0.004	214.998
3	217.000	−1.994	0.004	215.006
4	219.000	−3.947	0.004	215.053
Approximate value of unknown H_0 =				214.991

To compare the results the computations (the adjustment of observations and height calculation) will be done in a few variants:

- variant 1: the standard method of least squares (LSQ),
- variant 2: the EDF method (Figure 1b, equation 6),
- variant 3: the ELDF1 method (Figure 2, function f_1, equations 15, 17)
- variant 4: the ELDF2 method (Figure 2, function f_2, equations 16, 18)
- variant 5: the least-square method used with rejection of outliers (LSQ*).

4. Adjustment by EDF1 method

The detailed algorithm of the procedure will be shown on the example of the chosen method ELDF1, and then the results of all the methods would be compared. The models used in the least-square adjustment method are available in handbooks on adjustment computations [Wiśniewski 2005, Ghilani 2005].

Observation equations (standard and matrix notations)

$$h_i + v_i = H_0 + \delta H - R_i \tag{19}$$

where: H_o – approximate height of the determined point

$$\mathbf{V = AX - L} \tag{20}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot [\delta H] - \begin{bmatrix} 0 \\ 7 \\ 15 \\ 62 \end{bmatrix}_{(mm)}$$

Weights and weight matrix

$$p_i = \frac{1}{m_i^2} \tag{21}$$

$$\mathbf{P} = diag\{0.625;\ 0.625;\ 0.625;\ 0.625\}$$

Finding the unknown and residual vector (by LSQ method)

$$\mathbf{X} = (\mathbf{A}^{\mathrm{T}}\mathbf{PA})^{-1} \cdot \mathbf{A}^{\mathrm{T}}\mathbf{PL} = \delta H = 21\ \mathrm{mm}$$

$$\mathbf{V} = [21 \quad 14 \quad 6 \quad -41]^{\mathrm{T}}$$

Standardization of residuals (3), (4)

$$diag\{\mathbf{Q_v}\} = \{12;\ 12;\ 12;\ 12\}$$

$$\bar{\mathbf{V}} = [\{\bar{v}_i\};\ i = 1,\ \ldots,4]^{\mathrm{T}} = [6.06 \quad 4.04 \quad 1.73 \quad -11.84]^{\mathrm{T}}$$

It is assumed that the boundary criterion of damping function is $k = 6$ [Wiśniewski 2005, Gargula and Krupiński 2007]. Auxiliary parameters, according to (11) and (15) would have the value $k_o = 3$ and $k_r = 12$. Following (13), these parameters of a damping function would be checked with regard to what ranges belong the specific standardized residuals; on this basis a damping index $f(\bar{v}_i)$ would be determined. The calculations are made as an iteration process. The initial iteration ($j = 0$) includes results obtained by the standard LSQ method.

Iteration j = 0

$$\mathbf{V}^{(0)} = \mathbf{V};\ \mathbf{P}^{(0)} = \mathbf{P};\ \bar{v}_i^{(0)} = \bar{v}_i$$

Damping indices $\boldsymbol{f}(\bar{\boldsymbol{v}}_1)$ and damping function $\mathbf{F}\left(\bar{\mathbf{V}}^{(\mathbf{0})}\right)$

$$\bar{v}_1^{(0)} \in (k_o; k_r\rangle \rightarrow f(\bar{v}_1) = -\frac{0.5}{\sqrt{0.75} \cdot k} \cdot \bar{v}_1 + \frac{1}{\sqrt{0.75}} = 0.57$$

$$\bar{v}_2^{(0)} \in (k_o; k_r\rangle \rightarrow f(\bar{v}_2) = -\frac{0.5}{\sqrt{0.75} \cdot k} \cdot \bar{v}_2 + \frac{1}{\sqrt{0.75}} = 0.77$$

$$\bar{v}_3^{(0)} \in \langle -k_o; k_o\rangle \rightarrow f(\bar{v}_3) = \sqrt{1 - \frac{\bar{v}_3^2}{k^2}} = 0.96$$

$$\bar{v}_4^{(0)} \in (-k_o; -k_r\rangle \rightarrow f(\bar{v}_4) = \frac{0.5}{\sqrt{0.75} \cdot k} \cdot \bar{v}_4 + \frac{1}{\sqrt{0.75}} = 0.02$$

$$\mathbf{F}\left(\bar{\mathbf{V}}^{(0)}\right) = diag\{0.57 \quad 0.77 \quad 0.96 \quad 0.02\}$$

One can see that the observation h_4 (distinctly differing from the others) obtained very low damping index $f(\bar{v}_4) = 0.02$, which would mean practically zero weight in the next stage of adjustment.

Iteration $j = 1$

$$\mathbf{P}^{(1)} = \mathbf{F}\left(\bar{\mathbf{V}}^{(0)}\right) \cdot \mathbf{P}^{(0)} = diag\{0.036 \quad 0.048 \quad 0.060 \quad 0.0012\}$$

$$\mathbf{X}^{(1)} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}^{(1)}\mathbf{A}\right)^{-1} \cdot \mathbf{A}^{\mathrm{T}}\mathbf{P}^{(1)}\mathbf{L} = \delta H^{(1)} = 9.06$$

$$\mathbf{V}^{(1)} = \mathbf{A}\mathbf{X}^{(1)} - \mathbf{L} = [9.06 \quad 2.06 \quad -5.94 \quad -52.94]^{\mathrm{T}}$$

$$diag\{\mathbf{Q}_{\mathbf{V}}^{(1)}\} = diag\left\{\left(\mathbf{P}^{(1)}\right)^{-1} - \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}^{(1)}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\right\} = \{28.07 \quad 20.78 \quad 16.67 \quad 800\}$$

$$\bar{\mathbf{V}}^{(1)} = \left[\{\bar{v}_i^{(1)}\}; i = 1,\ldots,4\right]^{\mathrm{T}} = [1.97 \quad 0.55 \quad -1.90 \quad -1.88]^{\mathrm{T}}$$

All standardised residuals $\bar{v}_i^{(1)}$ obtained at this stage belong to the range $\langle -k_o\,;\ k_o \rangle$, and so the damping indices (diagonal elements of damping function) will be determined according to the function described by the equation (6):

$$\mathbf{F}\left(\bar{\mathbf{V}}^{(1)}\right) = diag\{0.94 \quad 1.00 \quad 0.95 \quad 0.95\}$$

Because the values of the matrix damping function are close to unity, subsequent iteration ($j = 2$) will introduce only insignificant changes in the results of the damping (at the level below 0.1 mm). Therefore the results obtained in this iteration ($j = 1$) will be used in the final adjustment of observations and of unknown (the height of a point).

5. The analysis of the results

According to the procedure outlined in the above section, the calculations were made with EDF function (6) and ELDF2 function (16). The final results are presented in Table 2.

Table 2. The results of the adjustment – comparison

The stage of adjustment	No.	LSQ	EDF	ELDF1	ELDF2	LSQ*
1	2	3	4	5	6	7
Initial observations h_i [m]	1	0.991	0.991	0.991	0.991	0.991
	2	−1.002	−1.002	−1.002	−1.002	−1.002
	3	−1.994	−1.994	−1.994	−1.994	−1.994
	4	−3.947	−3.947	−3.947	−3.947	–
Standardized residuals $\bar{v}_i$	1	–	6.06	6.06	6.06	–
	2	–	4.04	4.04	4.04	–
	3	–	1.73	1.73	1.73	–
	4	–	−11.84	−11.84	−11.84	–

Control parameters of the damping function	–	–	$K = 6$	$k = 6$ $k_o = 3$ $k_r = 12$	$k = 6$ $k_o = 4.2$ $k_r = 8.6$	–
Final weights p_i [m^{-2}]	1	0.0625	0.000	0.036	0.026	0.0625
	2	0.0625	0.046	0.048	0.046	0.0625
	3	0.0625	0.060	0.060	0.060	0.0625
	4	0.0625	0.000	0.001	0.000	–
Unknown δH [mm]	–	21.0	11.5	9.1	9.3	7.3
Final residuals v_i [mm]	1	21.0	11.5	9.1	9.3	7.3
	2	14.0	4.5	2.1	2.3	0.3
	3	6.0	−3.5	−5.9	−5.7	−7.7
	4	−41.0	−50.5	−52.9	−52.7	–
Adjusted unknown H_P [m]	–	215.0120	215.0025	215.0001	215.0003	214.9983
Adjusted observations $(h_i + v_i)$ [m]	1	1.0120	1.0025	1.0001	1.0003	0.9983
	2	−0.9880	−0.9975	−0.9999	−0.9997	−1.0017
	3	−1.9880	−1.9975	−1.9999	−1.9997	−2.0017
	4	−3.9880	−3.9975	−3.9999	−3.9997	–

LSQ – standard least-square method
EDF – adjustment method based on damping function proposed in [Gargula and Krupiński 2007, Gargula 2010]
ELDF1, ELDF2 – adjustment method based on damping function proposed in this study
LSQ* – standard least-square method after rejection of outlier h_4

The comparison of results allow to make a numerical analysis aimed at formulating the final conclusions as to the proposed damping function. The sample of observational data are chosen in such a way that one of the observation (h_4) is distinctly different than others in the context of the obtained adjustment results (the height of the junction point H_p – Figure 3).

The adjustment of observations according to the standard LSQ method (Table 2, column 3), as it can be easily seen, causes the dispersion of a gross error and weighs down on the final results (H_p = 215.0120). However, the analysis of final residual values opens up a possibility of identifying the suspected observation (v_4 = –41.0 mm), but the studied example applies to homogenous observations. In case of adjusting the observation systems of various types (angle, linear, elevation), identifying the outlier at this stage would be difficult.

Using the EDF function to adjustment of observation (column 4) leads to indication of standardized residuals values, which contain more precise information about the occurrence of gross error ($\overline{v}_4$ = –11.84). As a result, the observation is practically eliminated and obtains zero weight (numerically we ascribe to it the weight close to zero, e.g. 0.00001, because a diagonal element of a weight function cannot equal zero).

However the standard parameter value k = 6 adopted here [Gargula and Krupiński 2007] eliminates the observation h_1, because $\overline{v}_1$ = 6.06 (outside the accepted range). So the final result of the equation (H_p = 215.0025; almost 10 mm difference in comparison to LSQ) is probably closer to the real value, but its reliability is low – only 1 outlier.

The ELDF1 damping function (for an adopted criterion k_o = 0.5k, column 5) does not exclude any observation, but observation h_4 obtains very low weight (0.001) due to close proximity of its standardized residual (= –11.84) of a boundary value k_r = 12. However one intuitively expects that the outlier should be excluded from the adjustment process, so (for comparison purposes) the higher value of the control parameter was assumed: k_o = 0.7k (ELDF2 function, column 6). This adjustment enhances the damping impact (in comparison with ELDF1) on potential observations on standardized residuals within the range (Figure 2, equation 14). In this case the observation h_4 has been eliminated (the weight equals 0.000), but it did not have significant influence on the final result (H_p = 215.0003) in comparison with the ELDF1 method (H_p = 215.0001). It is related to the fact that already in the adjustment with the ELDF1 this observation obtained the weight close to zero (0.001).

The adjustment by the LSQ* method (after rejecting h_4) leads to a result (H_p = 214.9983) that differs by 2 mm from the result obtained by ELDF2 method. It should be remembered however that the ELDF2 function (like other damping functions: EDF, ELDF1) shows a different behavior than the function resulting from the standard LSQ method (see Figures 1a and 2).

6. Recapitulation and conclusions

This study proposed a way of using a new damping function in the algorithm of robust estimation. In the behavior of the damping function (ELDF) one can single out a fragment of conic curve (ellipse) and two tangents at points defined by the control parameter k_o and $-k_o$. The necessary mathematical equations have been introduced and a diagram of adjustment process of geodesic observations have been presented. The studied damping function was tested on two practical examples. To compare the effects of damping the outliers, an adjustment was done with the use of the author's damping function (EDF) and the standard least-square method.

The general conclusions are as follows:

- the ELDF method is effective in finding gross errors,
- the use of the ELDF function in robust estimation leads to elimination or reduction (damping) of gross error impact,
- in comparison with other functions (EDF, Hampel's etc.) the new function can gradually pass from the damping of outliers to their elimination from the adjustment process,
- apart from the standard parameter for standardized residuals (usually k = 6), the ELDF function has also the possibility of controlling (damping force) the additional parameter k_o in the suggested range $|k_o| \in (0.5k; k)$,

- the control (steering) parameters k, k_o can be adopted according to values proposed in this study or (with greater experience) their optimal values can be determined empirically.

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